(Pages : 3) **10616**

Reg. No. : ...............................

Name : ...................................

# Fourth Semester B.Tech. Degree Examination, February 2016
## (2013 Scheme)
## 13.401 : ENGINEERING MATHEMATICS – III (BCHMNPSU)

Time : 3 Hours Max. Marks : 100

### PART – A

Answer **all** questions. **Each** question carries **4** marks.

1. If f(z) and $\overline{f(z)}$ are analytic, then prove that f(z) is a constant.
2. Define critical point and invariant point of a transformation. Find the critical points and invariant points of $w = \frac{5-4z}{4z-2}$.
3. Evaluate $\int_C |z|\bar{z}dz$ where C consists of the upper simi circle | z | = 1 and the segment $-1 \le x \le 1$.
4. Solve $x = \sqrt[3]{25}$ by Newton-Raphson method, correct to three places of decimals.
5. Using Taylor's series method, find y to five places of decimals when x = 1.3, given that $dy = (x^2y-1)dx$ and y = 2 when x = 1.

### PART – B

Answer **one full** question from **each** Module. **Each** question carries **20** marks.

**Module – I**

6. a) Prove that $f(z) = \frac{x^3(1+i) - y^3(1-i)}{x^2+y^2}$, $z \neq 0$ and f(0) = 0 is continuous and satisfies CR equations at the origin, but f'(0) does not exist.

   b) Find the analytic function f(z) = u + iv if $u = e^{-x}(2xy\cos y + (y^2 - x^2)\sin y)$.

   c) Discuss the transformation $w = z + \frac{1}{z}$.

P.T.O.

7. a) Show that $u(x, y) = x^2 - y^2$ and $v(x, y) = \frac{-y}{x^2 + y^2}$ are both harmonic, but $u + iv$ is not analytic.

b) Determine the region of the w-plane into which the triangular region bounded by $x = 1$, $y = 1$ and $x + y = 1$ is mapped by $w = z^2$.

c) Find the bilinear transformation which maps $(2, i, -2)$ into the points $(1, i, -1)$.

## Module – II

8. a) Evaluate $\int_{|z|=3} \frac{e^z}{(z-2)(z+1)^2(z-1)} dz$ using Cauchy's integral formula.

b) Find the Laurent's series of $\frac{7z-2}{z(z+1)(z-2)}$ in $1 < |z+1| < 3$.

c) Show that $\int_0^\infty \frac{dx}{1+x^6} = \frac{\pi}{3}$.

9. a) Evaluate $\int_{|z|=1} z^4 e^{1/z} dz$.

b) Evaluate $\int_0^\pi \frac{a}{a^2 + \sin^2\theta} d\theta$.

c) Expand $ze^z$ about $z = 1$.

## Module – III

10. a) Find a real roof of the equation $3x - \cos x - 1 = 0$ which lies between 0 and 1 by Regula Falsi method.

b) By means of Lagrange's interpolation formula, prove that $y_3 = 0.05(y_0 + y_6) - 0.3(y_1 + y_5) + 0.75(y_2 + y_4)$.

c) Solve by Gauss – Seidel iteration method $10x - 2y - z - w = 3$, $-2x + 10y - z - w = 15$, $-x - y + 10z - 2w = 27$, $-x - y - 2z + 10w = -9$.

11. a) Find a real root of the equation $x^3 - x^2 + x - 7 = 0$ lying between 2 and 3 using bisection method correct to three decimal places.

b) From the following table, find the number of students who obtained marks in mathematics between 40 and 70.

| **Marks :** | 35 – 45 | 45 – 55 | 55 – 65 | 65 – 75 | 75 – 85 |
|---|---|---|---|---|---|
| **No. of Students :** | 18 | 40 | 64 | 50 | 28 |

c) By Gauss elimination method, solve
$x + y + 2z - w = 5$, $x + 3y + 2z + w = 17$, $x + y + 3z + 2w = 20$,
$x + 3y + 4z + 2w = 27$.

**Module – IV**

12. a) Find the approximate value of $\int_0^4 \sqrt{64 - x^2}\, dx$ by (i) Trapezoidal rule (ii) Simpson's rule, by taking $h = 0.5$.

b) Solve $\frac{dy}{dx} = x - 2y$ for $x = 0.2$ by using RK method. Initial values are $x = 0$, $y = 1$ and $h = 0.1$.

13. a) Solve the Laplace's equation $u_{xx} + u_{yy} = 0$ in the square region $0 \le x \le 4$, $0 \le y \le 4$ with boundary conditions $u(0, y) = 0$, $u(4, y) = 8 + 2y$, $u(x, 0) = 2x$ and $u(x, 4) = x^2$ with $h = 1$.

b) Use modified Euler's method to find the value of $y(1.1)$ when $\frac{dy}{dx} = x + xy$, $y(1) = 1$, taking $h = 0.05$.